Game Theoretical Analysis of Blockchain Users' Behaviors

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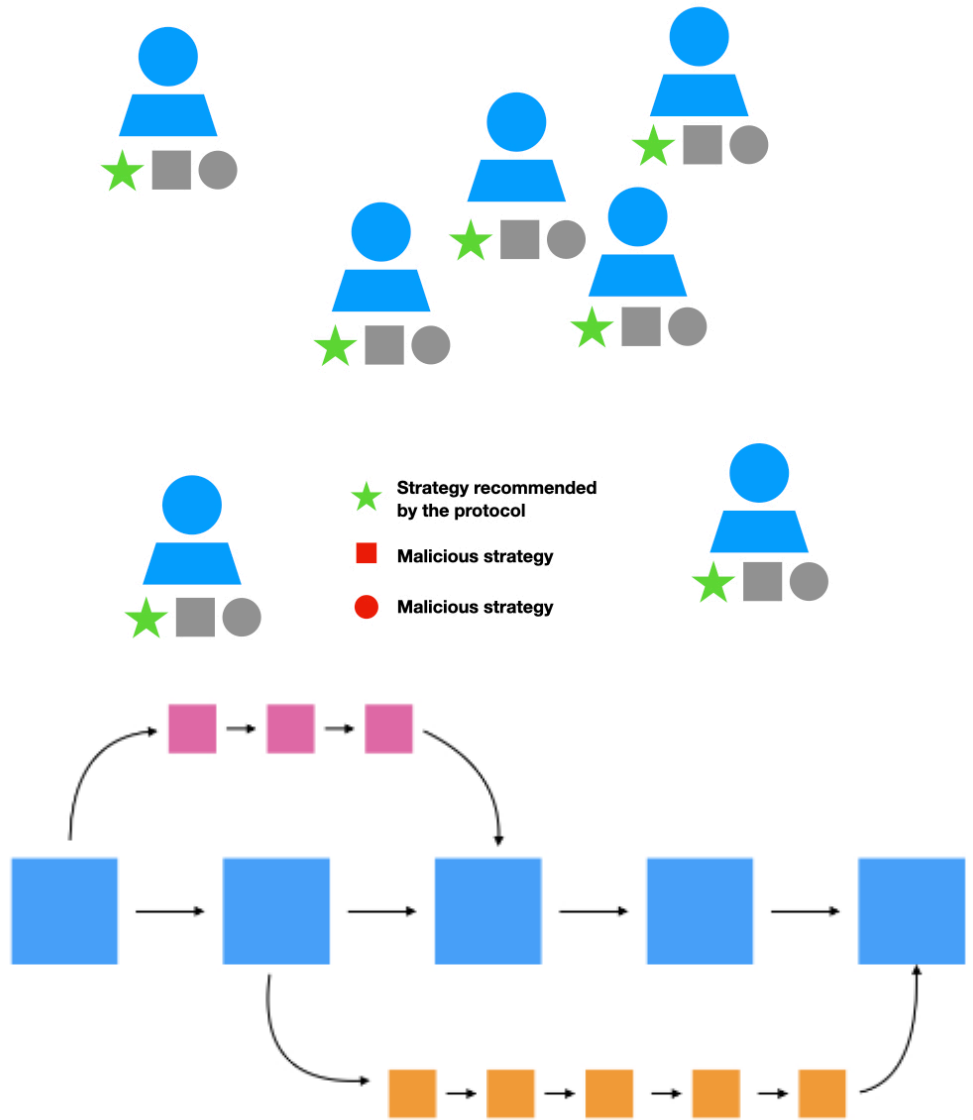
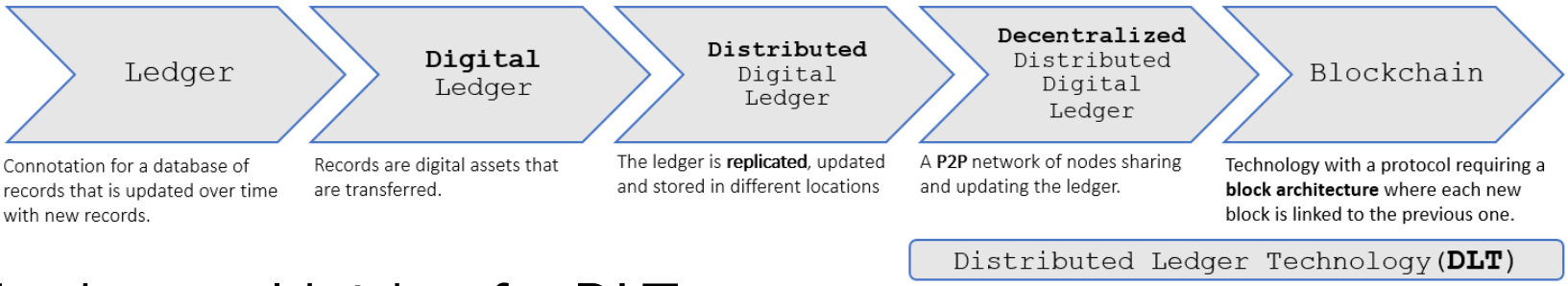
THÈSE


Strategy recommended by the protocol
Malicious strategy
Malicious strategy

Introduction on DLTs



Fundamental bricks of a DLT:

- **Data Structure** (*e.g., Bitcoin UTXO*)
- **Communication Language** to update the ledger state (*e.g., transactions and smart-contract*)
- **Agreement Protocol** (*e.g., Bitcoin proof-of-work*)

Blockchain Actors and Transaction Journey

Transacting parties: a blockchain transaction involves two different types of actors related to single or multiple blockchain users: the *data-sender* and the *data-receiver*.

Validating nodes: run the consensus algorithm and are responsible for establishing the agreement on the proposals made by other validators or by leading nodes.

1. **Transaction Creation**
   - Signing transactions
2. Transaction Propagation
   - Collect transactions in blocks
4. **Transaction Validation**
5. Transaction Confirmation

Overview

Goal: We analyze the two main types of blockchain users (i.e., transacting parties and validating nodes) as well as different blockchains (i.e., permissionless and permissioned) with the scope of providing a general overview of the topic and formal results on **blockchain users' behaviors**.

		Behavior Model	
		Rational	*Byzantine*
Blockchain User	*Validating nodes*	*Miners*	*Robustness layer-1 blockchains*
	Transacting parties	*Interoperbility*	*Robustness layer-2 blockchains*

Rationality and Games

Pillars of game theory:

1. Agents **rationality**: players are fully able to analyze the problem (i.e., the game) and take the right decisions.
2. Each player is able to **order** the outcomes of the game according to consistent preferences.

Game $\longrightarrow$ blockchain consensus protocol, mining game, swap protocol
Players $\longrightarrow$ validating nodes, transacting parties
Strategy $\longrightarrow$ follow the instructions of a protocol or deviate
Payoff $\longrightarrow$ value of the final outcome (utility function)

How players play

- **Non-Cooperative Games**: individually as solo players
- **Cooperative Games**: in groups forming coalitions

Definitions

Definition (Non-cooperative or Strategic Game)

A game in a normal form representation is identified by a tuple $\boxed{\Gamma = \langle N, \mathcal{S}, u \rangle}$, where

- N is a finite set of n players
- $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \cdots \times \mathcal{S}_n$ where $\mathcal{S}_i$ is the set of strategies of player i
- $u : \mathcal{S} \rightarrow \mathbb{R}^n$ is the utility function of the players.

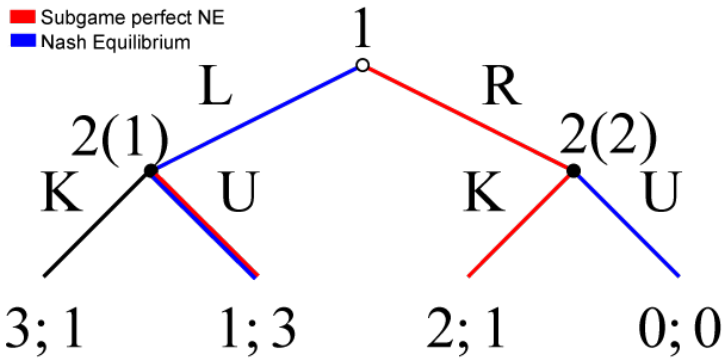
Definition (Nash equilibrium)

A strategy profile $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_n) \in \mathcal{S}$ is a Nash equilibrium if for every player i and for every strategy $\tau_i \in S_i$ we have that :

$$u_i(\sigma_1, \sigma_2, \ldots, \sigma_i, \ldots, \sigma_n) \geq u_i(\sigma_1, \sigma_2, \ldots, \tau_i, \ldots, \sigma_n)$$

Game Representations

- **Sequential games**: players play one after the other
- **Normal form games**: players' actions do not have an order (can take place simultaneously)

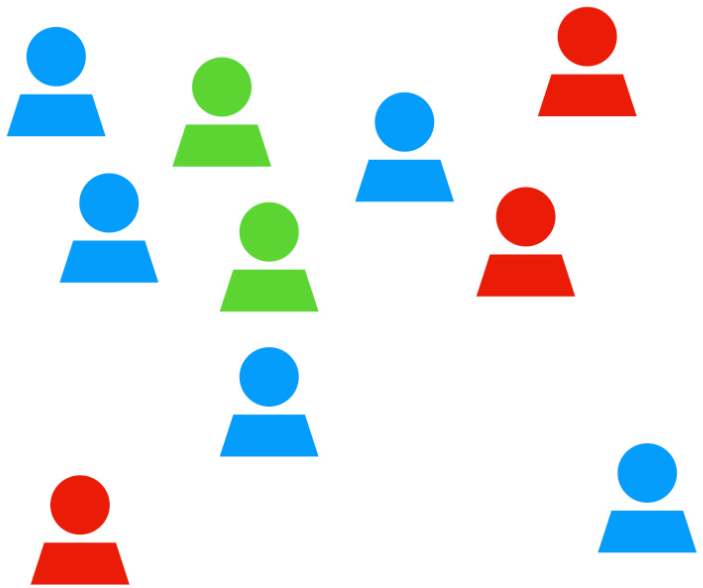


	(K,K)	(K,U)	(U,U)	(U,K)
L	3,1	3,1	1,3	1,3
R	2,1	0,0	0,0	2,1

BAR: Rational, Altruistic and Byzantine

Both blockchain users are at first modeled as **rational agents**. This game-theoretical modeling enables to capture several behaviors. To be more generic (i.e., including also irrational or unexpected malicious behaviors) blockchain users are modeled also as **Byzantine agents**. Users can be split into 3 categories:

- Byzantine, when they deviate arbitrarily from the protocol
- Altruistic, when they follow the protocol
- Rational, when they act to maximize their utility function



Overview

1. Belotti et al. "A vademecum on blockchain technologies: When, which, and how." IEEE Communications Surveys Tutorials 21.4 (2019): 3796-3838.
2. Belotti et al. "Bitcoin pool-hopping detection." 2018 IEEE 4th International Forum on Research and Technology for Society and Industry (RTSI). IEEE, 2018.
3. Belotti et al. "Rewarding miners: bankruptcy situations and pooling strategies." Multi-Agent Systems and Agreement Technologies. Springer, Cham, 2020. 85-99. ⟵ *Bankruptcy (cooperative) games*
4. Belotti et al. "Game theoretical analysis of atomic cross-chain swaps." 40th IEEE International Conference on Distributed Computing Systems (ICDCS). 2020. ⟵ *Strategic games, NE*
5. Zappalà, Belotti et al. "Game theoretical Framework for Analyzing Blockchains Robustness." International Symposium on Distributed Computing (DISC). 2021. ⟵ *Strategic games, NE*

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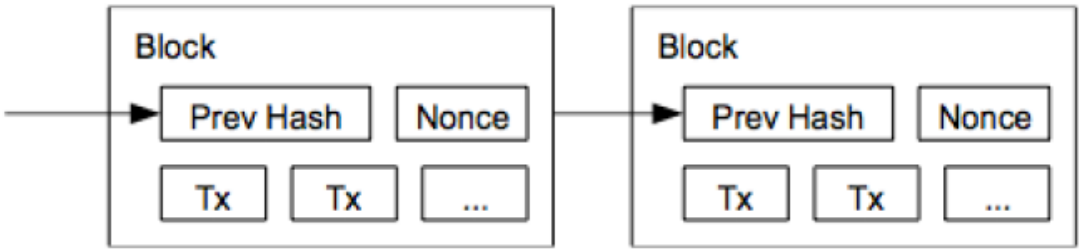
Rational Miners

How to create bitcoins? Mining



SHA256(input, nonce) $\leq$ difficulty target

nonce = **full solution** and **share** = next to valid solution.
Pools' rewarding systems based on the number of **reported** shares.

The rewarding method adopted by a pool take into account that:

- Miners are **rational** agents.
- *Strategic behaviors:* WHEN reporting shares & WHERE directing the effort.

- Division into **rounds**: time elapsing between 2 full solutions.
- Reward allocation: $R : \mathrm{s} \rightarrow [0,1]^n$ where $\mathrm{s} \in \mathbb{N}^n$ vector of total number of shares reported by each miner in a **round**.

Original rewarding methods

Two types of malicious behaviors are analyzed; *pool-hopping* and *block-withholding*.

Original Rewarding Methods:

1. **proportional**: $R_i(\mathbf{s}) = \frac{s_i}{||\mathbf{s}||_1}$.
2. **pay-per-share**: $R_i(\mathbf{s}) = \frac{s_i}{D}$.

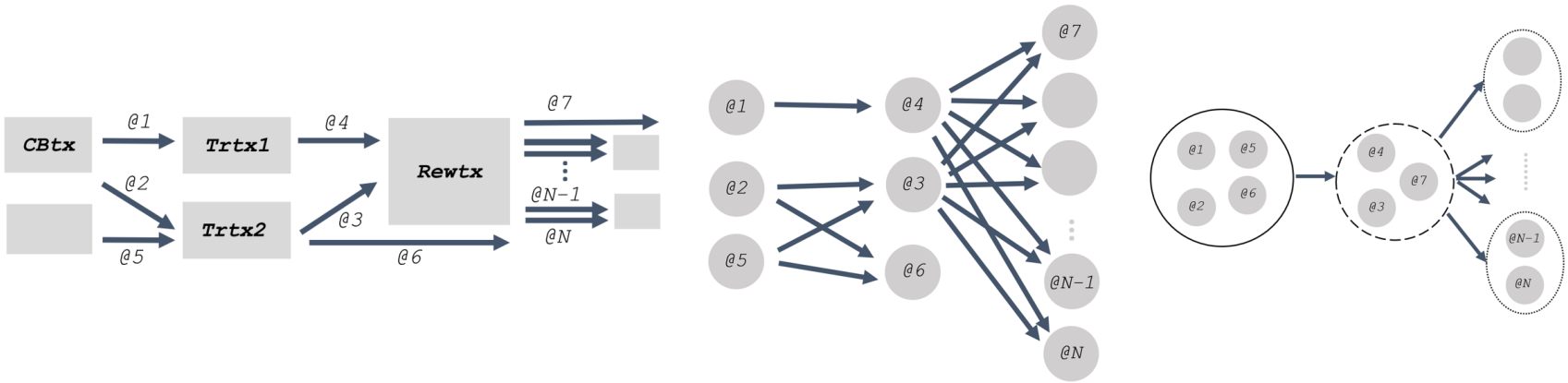
Issues:

- **Favoring Pool-Hopping** –> change pool during the round.
  The longer the round, the less each share is worth
- **Block Withholding** –> Pools go bankruptcy.
  The longer the round, the more each share is worth

Investigating the Network

Coinbase transactions, *Rewarding transactions*, *Transferring transactions*

- Identify miners –> **User sub-network** of a *subset* of transactions.
- Identify potential hoppers –> Filtering rewarding transactions.
- Identify real hoppers –> Transactions **reordering**.

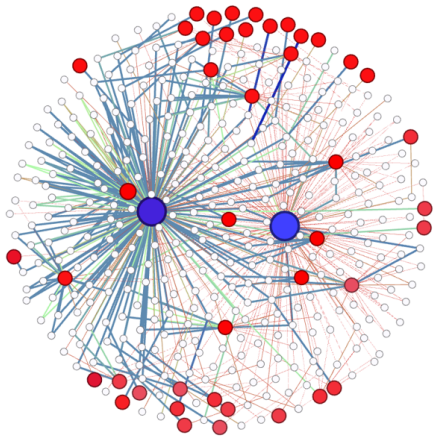


Transaction network –> **Address** network –> **User** network

mapping procedure: basic heuristics applied to all network addresses.

Results

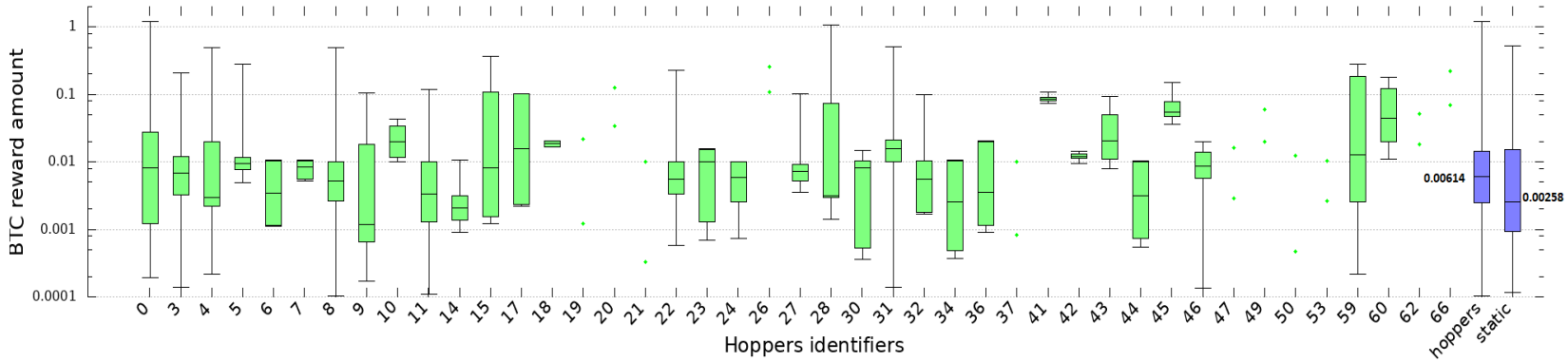
2-pool: Slush and Kano pool (*slush* and *pplns* methods) **time period**: April 6-20, 2016



Left blue: Kano pool
Right blue: Slush pool
Red nodes: detected hoppers

Edge **thickness** and **color** depend on the frequency of users interactions.

higher gains hopping i.e., earn on average more than the default miners.



Incentive Compatible Rewarding Function

Division into *rounds* where $||\mathbf{s}||_1$ shares are submitted.

1. **proportional**: $R_i(\mathbf{s}) = \frac{s_i}{||\mathbf{s}||_1}$.
2. **pay-per-share**: $R_i(\mathbf{s}) = \frac{s_i}{D}$.

Proposition

(**Incentive Compatibility**) A reward function R is *incentive compatible* when every miner's best response strategy $\sigma(R)$ reports full solutions immediately.

$$R_i^{(ic)}(\mathbf{s}, w) = \begin{cases} \frac{s_i}{D} + \mathbf{1}_{\{i=w\}}\left(1 - \frac{||\mathbf{s}||_1}{D}\right), & \text{if } ||\mathbf{s}||_1 < D \\ \frac{s_i}{||\mathbf{s}||_1}, & \text{if } ||\mathbf{s}||_1 \geq D \end{cases}$$

where w is the discoverer of the full solution.

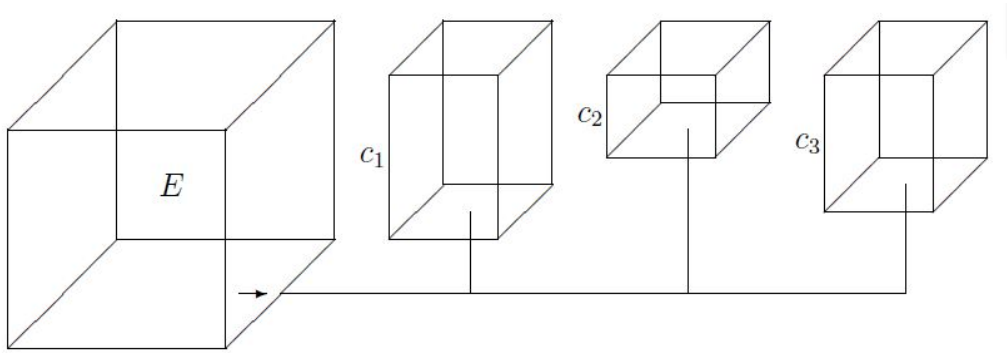
Reinterpretation as an outcome of a bankruptcy game

Bankruptcy Games - CEL

Definition

A *bankruptcy game* on the set N consists of a pair $(\mathbf{c}, E) \in \mathbb{R}^N \times \mathbb{R}$ with $c_i \geq 0$ for all $i \in N$ and $0 < E < \sum_{i \in N} c_i = C$. For each bankruptcy problem $(\mathbf{c}, E) \in \mathbb{B}^N$,

- **Proportional** rule **(P)**:
  $f(\mathbf{c}, E) = \lambda \mathbf{c}, \qquad \lambda : \sum_{i \in N} \lambda c_i = E$
- **Constrained equal losses** rule **(CEL)**:
  $f(\mathbf{c}, E) = \max\{c_i - \lambda, 0\}, \quad \lambda : \sum_{i \in N} \max\{c_i - \lambda, 0\} = E$



idea: CEL can prevent pool-hopping.

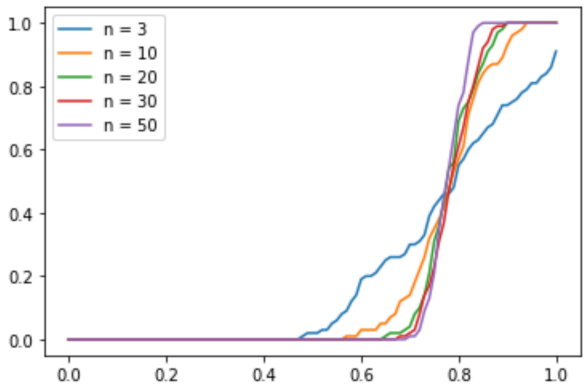
New Incentive Compatible Reward Function

$$\widehat{R}_i(\mathbf{s}, w) = \begin{cases} \frac{s_i}{D} + \mathbf{1}_{\{i=w\}}\left(1 - \frac{||\mathbf{s}||_1}{D}\right), & \text{if } ||\mathbf{s}||_1 < D \\ \frac{\mathbf{1}_{\{i=w\}}}{D} + \max\left(\frac{s_i}{D} - \lambda, 0\right) \quad \lambda : \text{efficiency}, & \text{if } ||\mathbf{s}||_1 \geq D \end{cases}$$

Proposition

The reward function R proposed by *Schrijvers et. al.* always gives miners a positive incentive $\delta_{\text{hop}} > 0$ to perform pool hopping. CEL-based reward function gives less incentive to pool hopping.

Estimate percentage $p(\alpha)$ non-hoppers and simulate its values.

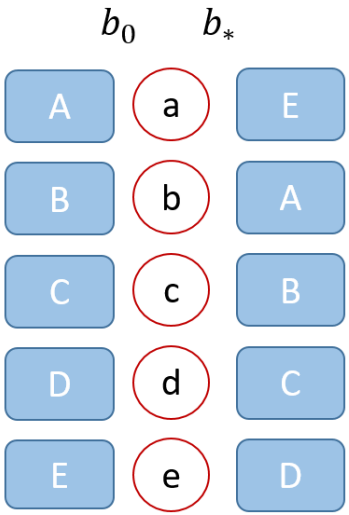


Swap Problem

Rational Agent: Rational Transacting Parties

A ***swap problem*** is a tuple $\boxed{\langle \mathcal{A}, \mathcal{O}, b_0, b_*, (u_i)_{i\in\mathcal{O}} \rangle}$ where:

- $\mathcal{A} = \{1, \ldots, m\}$ is the set of *assets*;
- $\mathcal{O} = \{1, \ldots, n\}$ is the set of *owners* or *agents*, with $m \geq n$;
- $b_0, b_* : \mathcal{A} \to \mathcal{O}$ (both **surjective**) the *original* and the *desired* ownership map;
- u_i is the payoff function for owner $i \in \mathcal{O}$ over bundles of assets in $2^{\mathcal{A}}$ such that $u_i(b_0^{-1}(i)) < u_i(b_*^{-1}(i))$ and for any $S, T \in 2^{\mathcal{A}}$ with $S \subseteq T$ we have $u_i(T) \geq u_i(S)$, $\forall i \in \mathcal{O}$.



Step 1: Decentralized Exchange protocols

Step 2: Decentralized Swap protocols

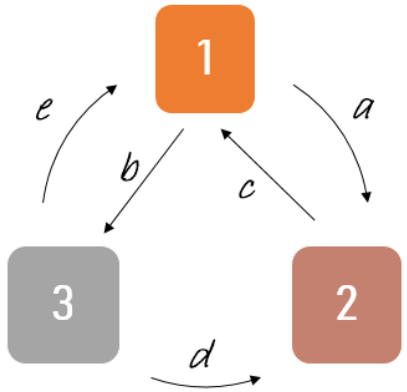
Step 3: Blockchain Swap protocols

Decentralized Exchange Protocols

A sequence $\boxed{\sigma = \{(A^k, O^k, X^k) : |A^k| \geq |O^k|\}_k}$, $k \in \{1, \ldots, t\}, t \in \mathbb{N} : t \leq m$ defines a **decentralized exchange protocol** where:

- $A^k \subseteq \mathcal{A}$ asset involved in the exchange at step k;
- $O^k \subseteq \mathcal{O}$ owners involved in the exchange at step k;
- $X^k : A^k \to O^k$ (surjective) specifies the owner $X^k(a) \in O^k$ of any asset $a \in A^K$ at step k;

A *decentralized swap protocol* is a decentralized exchange protocol where $\{A^k : k = 1, \ldots, t\}$ is a partition of $\mathcal{A}$.



Example. $\mathcal{A} = \{a, b, c, d, e\}, \mathcal{O} = \{1, 2, 3\}$, $b_0 = (1, 1, 2, 3, 3)$ and $b_* = (2, 3, 1, 2, 1)$.

$\sigma = (\{a, c\}, \{1, 2\}, \{X^1(a) = 2, X^1(c) = 1\}), (\{b, e\}, \{1, 3\}, \{X^2(b) = 3, X^2(e) = 1\}), (\{d\}, \{2\})$.

Decentralized Blockchain Swap Protocols

Assets involved in the swap should be ***locked*** for the following reasons:

(i) consequently to failures in the assets locking, the initial situation must be restored;

(ii) once an asset transfer is committed all the other transfers have to be committed, too.

Any transfer should be conditioned on the correct asset locking.

A **decentralized blockchain swap protocol** is defined by the pair $\boxed{(\sigma_P, \sigma_T)}$ where

- $\sigma_P = \{(A^j, O^j)\}_{j \in \{1,\dots,t_P\}}$, $t_P \in \mathbb{N} : t_P \leq m$, $A^j \subseteq \mathcal{A}, O^j \subseteq \mathcal{O}$ is a sequence such that $\forall j \in \{1, \dots, t_P\}$, $O^j = \{o \in \mathcal{O} : o \in b_*(A^j) \vee o \in b_0(A^j)\}$;
- $\sigma_T = \{(A^k, O^k, X^k)\}_{k \in \{1,\dots,t_T\}}$ is a swap protocol engendering the sequence of maps $b_1^{\sigma_T}, \dots, b_{t_T}^{\sigma_T} : \mathcal{A} \to \mathcal{O}$.

Strategic and Extensive form Games

Definition

A *decision function* as a map $F : \{1, \ldots, t\} \to \mathcal{O} \cup \mathcal{T}$ that specifies which owner $F(k)$ has the power to decide at step k whether to transfer A^k to O^k. F_T is **effective** on σ_T iff $F_T(k) = O^k$ for any $k \in \{1, \ldots, t_T\}$.

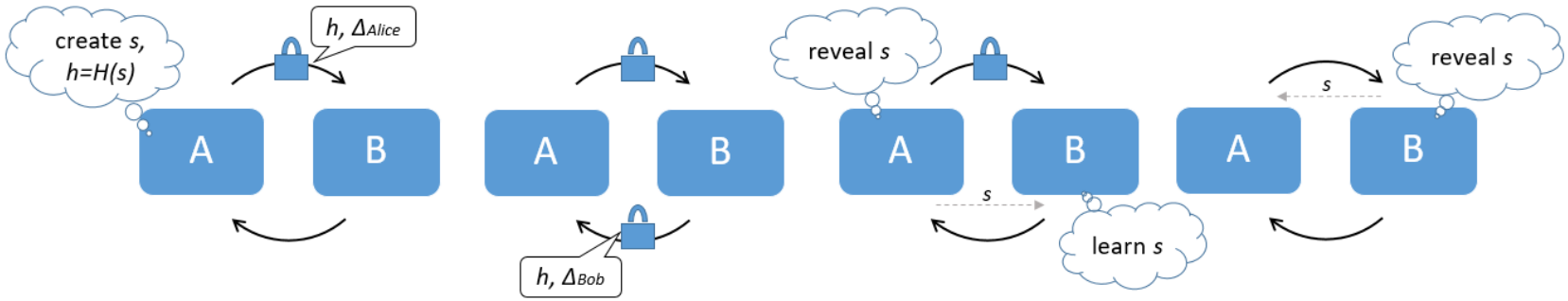
1. Swap protocols with *sequential publishing* and *commitment*.
2. Swap protocols with *concurrent publishing* and *snap commitment*.

1. **Extensive games**: sequential phases.
2. **Normal form games**: concurrent phase.

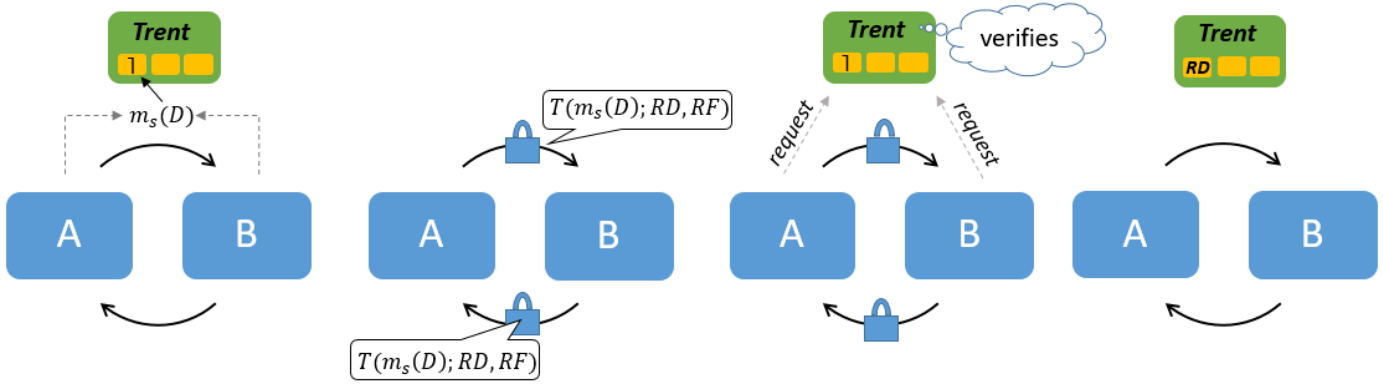
Strategies:

- **Follow**: each player follow the protocol in every step.
- **Deviate**: the player decide to behave *irrationally* or *maliciously* and decide not to publish or not to trigger a transaction.

Sequential (*Nolan*) and Concurrent-Snap protocols



Alice swaps x Acoins for y Bcoins owned by Bob.Nolan's first protocol for UTXO-based blockchains (**not atomic**).
$\sigma_P = \{(x, B), (y, A)\}, F_P(j) = \{A, B\}, j = \{1, 2\}; \sigma_T = \{(y, A), (x, B)\}, F_T(k) = \{A, B\}, k = \{1, 2\}.$

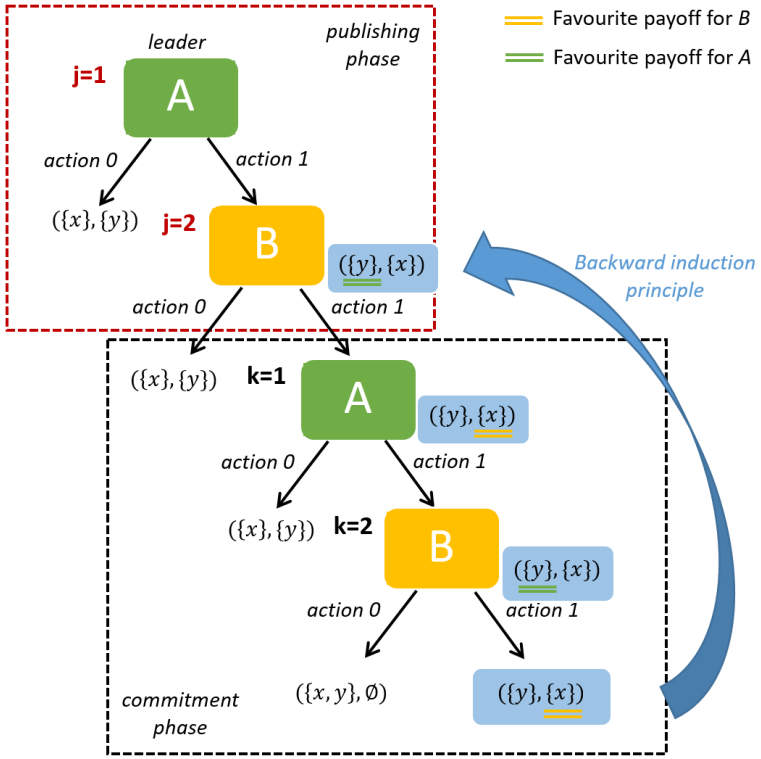


Atomic protocol with central authority. $\sigma_P = \{(\{x, y\}, \{A, B\})\}, F_P(j) = \{A, B\}, j = \{1\};$
$\sigma_T = \{(\{x, y\}, \{A, B\}, \{X^1(x) = B, X^1(y) = A\})\}, F_T(k) = T, k = \{1\}.$

Results

Proposition

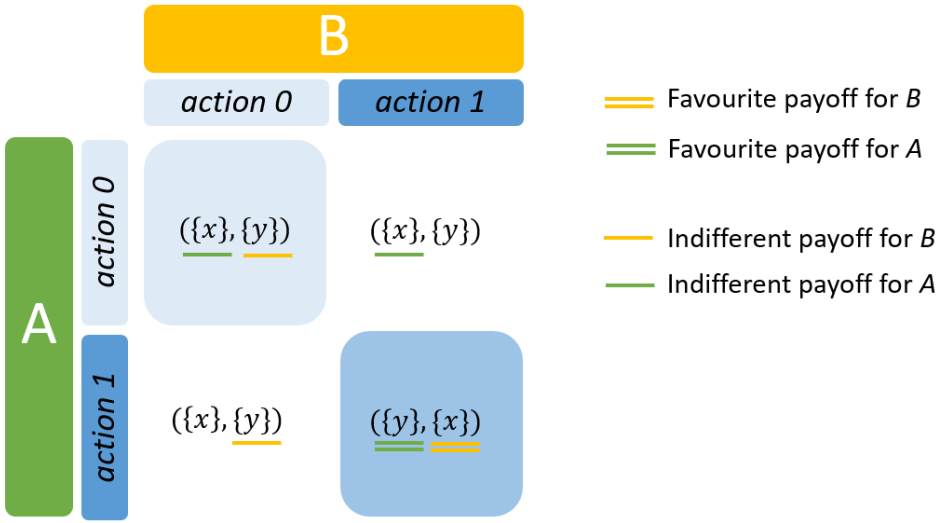
Let Γ^{σ} be the extensive form game associated with a sequential swap problem, if F is **effective**, then the strategy profile $(\hat{s}_1, \ldots, \hat{s}_n)$ that specifies action 1 (follow the protocol) at any node is the **unique subgame perfect equilibrium**.



Results - part 2

Proposition

Let Γ be the strategic form game associated with a swap problem characterized by a *concurrent publishing and a snap commitment* where the decision function F_T is such that $F_T(k) = T \in \mathcal{T}\ \forall k \in \{1, \ldots, t_T\}$. Then, the strategy profile $(\hat{s}_1, \ldots, \hat{s}_n)$ that specifies action 1 for every player i is a **Nash equilibrium**.



Goal and Approach

- Byzantine, when they deviate arbitrarily from the protocol
- Altruistic, when they follow the protocol
- Rational, when they act to maximize their utility function

		Behavior Model	
		Rational	*Byzantine*
Blockchain User	*Validating nodes*	*Miners*	*Robustness layer-1 blockchains*
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Goal. **Characterize** and **measure** in a formal and general way the robustness of a blockchain protocol.
Approach.

- List all the protocol instructions
- Identify all the possible behaviors of the users when facing the protocol
- List and evaluate all the possible outcomes

Model - Mechanism and Robustness Properties

Definition

A game in a normal form representation is identified by a tuple $\Gamma = \langle N, \mathcal{S}, u \rangle$.
A **mechanism** is a pair $\boxed{(\Gamma, \sigma)}$ in which $\Gamma = \langle N, \mathcal{S}, u \rangle$ is a game and $\sigma \in \mathcal{S}$ is a joint strategy.

We fix the **maximum number of rational players allowed in a game such that the protocol still provides the same utility to altruistic players**.

Property

A joint strategy $\sigma \in \mathcal{S}$ is a **k-resilient** equilibrium if for all $C \subseteq N$ with $1 \leq |C| \leq k$, all $\tau_C \in \mathcal{S}_C$ and all $i \in C$, we have $u_i(\sigma_C, \sigma_{-C}) \geq u_i(\tau_C, \sigma_{-C})$.

Model - Mechanism and Robustness Properties

We fix the **maximum number of byzantine players allowed in a game such that the altruistic players always get at least the utility of the initial state**.

Property

A joint strategy $\sigma \in \mathcal{S}$ is **t-weak-immune** if for all $T \subseteq N$ with $|T| \leq t$, all $\tau_T \in \mathcal{S}_T$ and all $i \in N \setminus T$, we have $u_i(\sigma_{-T}, \tau_T) \geq 0$.

Definition

Given 2 games A, B with the same set of players N it is possible to define a new game $\boxed{C = A \odot B}$, called **composition** of A and B, which is characterized as follows.
$C = \langle N, (s_{Ai}, s_{Bi}), u_A + u_B \rangle$.

The compositions preserves the robustness properties

Results

Here some **indices** to measure robustness of *layer-1* (users = **validating nodes**) and *layer-2* (user = **transacting parties**) blockchain protocols

Protocol	k-Resilience	t-Weak Immunity
Tendermint	**Yes, k** $< n/3$	**Yes, t** $< n/3$
Bitcoin	**Yes, k** $< 3n/20$	**No**
Lightning Network	**Yes, k** $< 3n/20$	**No**
Closing module	Yes	No
(Alternative closing module)	(Yes)	(Yes)
Other modules	Yes	Yes
Side-chain (Platypus)	**Yes, k** $< n/3$	**Yes, t** $< n/3$
Cross-chain Swap	**Yes**	**Yes**

Present Works

1. Belotti et al. "A vademecum on blockchain technologies: When, which, and how." IEEE Communications Surveys Tutorials 21.4 (2019): 3796-3838.
2. Belotti et al. "Bitcoin pool-hopping detection." 2018 IEEE 4th International Forum on Research and Technology for Society and Industry (RTSI). IEEE, 2018.
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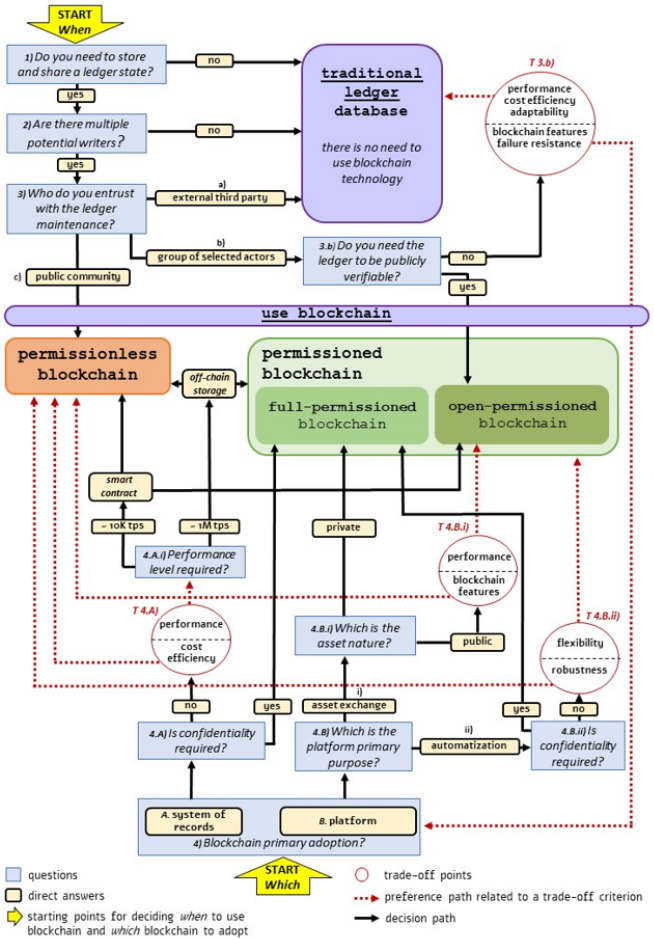
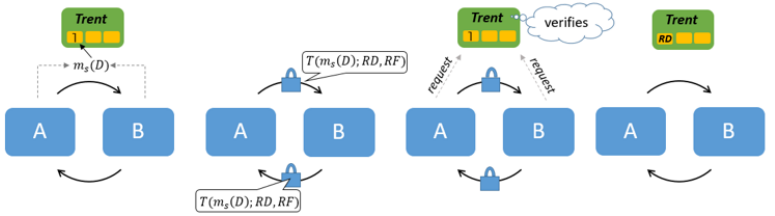
5. Zappalà, Belotti et al. "Game theoretical Framework for Analyzing Blockchains Robustness." International Symposium on Distributed Computing (DISC). 2021.

Present Works - Industrial Side

User as **early adopter** facing several challenges: using the technology, respecting the regulation and which role to assume in a blockchain protocol.





Future Works

Future working directions.

1. **Rational Behaviors**. modeling blockchain systems with fewer assumptions.
2. **Solution Concepts**. analysis of blockchain users' behaviors with other solution concepts.

3. **Blockchain Users' Behaviors**. extend analysis not only to new blockchain protocols but also to include new protocol deviations.
4. **Rewarding Miners**. applying the analysis to other crypto-currencies with comparable and new consensus methods.

Thanks for the Attention